1.4 Inclination and Slope

Pre-Requisite Knowledge:

Mathematical convention from Trigonometry - all angles measured in the counterclockwise direction are positive. Those measured in the clockwise direction are negative. We use θ to represent an angle.

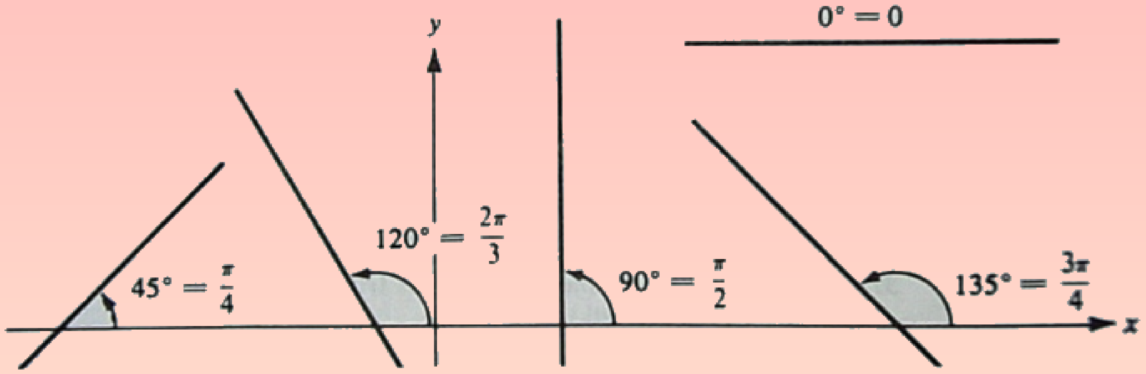


Figure 1.23

DEFINITION

The inclination of a line that intersects the x axis is the measure of the smallest nonnegative angle which the line makes with the positive end of the x axis. The inclination of a line parallel to the x axis is 0.

DEFINITION

The inclination of a line is always less than 180° or π radians and every line has inclination. Thus for every line $0 \leq \theta < 180^\circ$ or $0 \leq \theta < \pi$ given that θ represents inclination.

We cannot use inclination to find a simple relationship between a lines coordinates and the inclination without having to resort to trigonometric function. Thus we have to consider another expression related to the inclination of a line, namely slope.

DEFINITION

The slope m of a line is the tangent of the inclination; thus,

$$m = \tan\theta$$

While it is possible for two different angles to have the same tangent, two different lines cannot have the same slope because we restricted inclination to $0 \le \theta < 180°$. Using $m = \tan\theta$ we have a special situation when $\theta = 90°$. Because tan 90° does not exist we say that vertical lines have an inclination of 90° but no slope. It is important to note that "no slope" is different from "zero slope." For all other situations we are able to rely upon the definition of the tangent function.

$$\tan\theta = \frac{y}{x}.$$

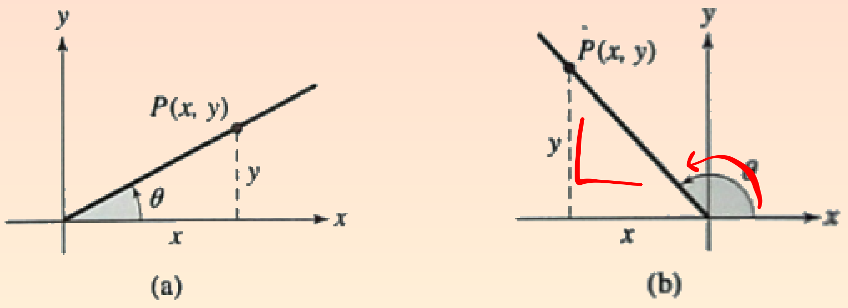
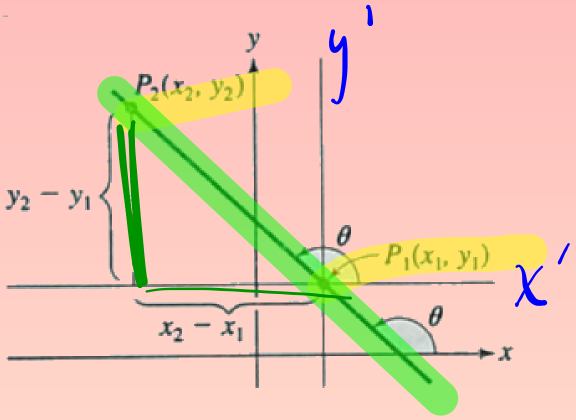


Figure 1.24

Example 1: Derive the traditional slope formula



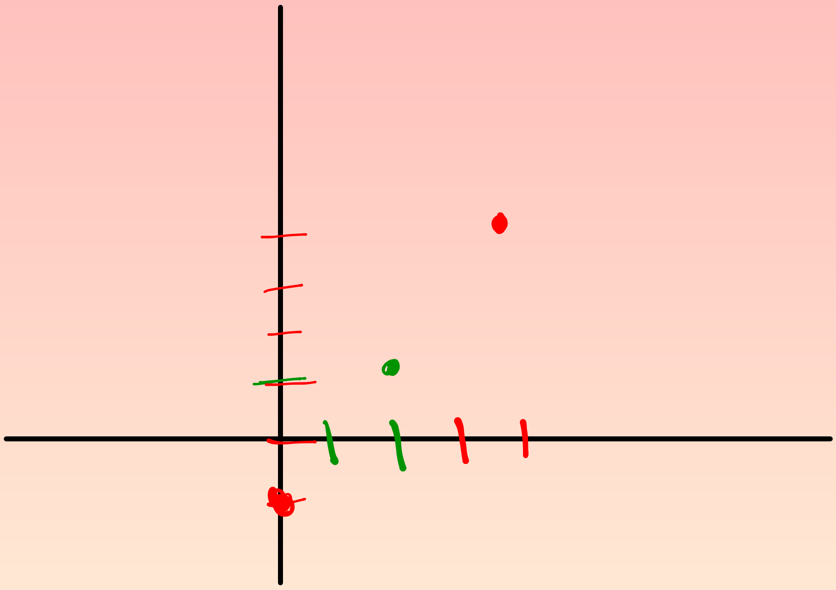
$m = \tan \theta$

$= \frac{y}{x}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Theorem 1.4

Example 2: Plot the line through (2, 1) with slope 3/2.



1.5 Parallel and Perpendicular Lines

Parallel:

If two non vertical lines are parallel, then they must have the same inclination and, thus, the same slope. If two parallel lines are vertical, then neither one has slope. Similarly, if $m_1 = m_2$ or if neither line has slope, then the two lines are parallel. Thus two lines are parallel if and only if $m_1 = m_2$ or if neither line has slope.

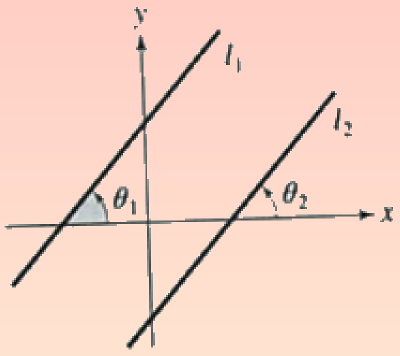
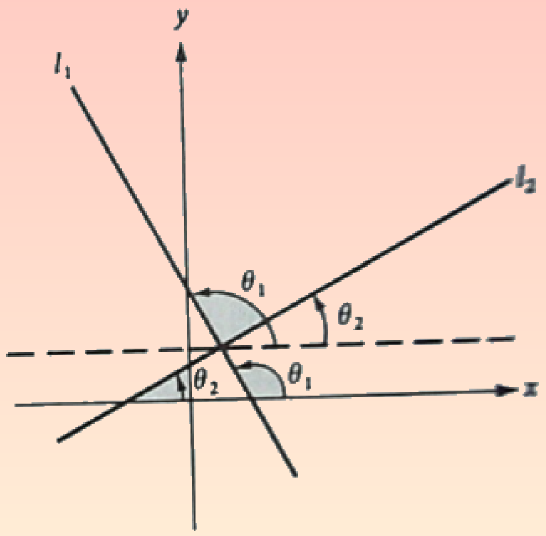


Figure 1.30

Perpendicular:

If two non vertical lines l_1 and l_2 with the respective inclinations θ_1 and θ_2 are perpendicular then $\theta_1 = \theta_2 + 90^\circ$. Thus applying the definition of slope tan θ_1 = tan $(\theta_2 + 90^\circ)$ = $-\cot(\theta_2)$ = $- 1/(\tan(\theta_2))$. Thus $m_1 = -1/m_2$.



Theorem 1.5

The lines l_1 and l_2 with slopes m_1 and m_2, respectively, are

(a) parallel or coincident if and only if $m_1 = m_2$,

(b) perpendicular if and only if $m_1 m_2 = -1$.

Example 3: Determine if l_1 containing the points (1,5) and (3, 8) is parallel, coincident, perpendicular, or neither to l_2 which contains the points (-4, 1) and (0, 7)

$m_1 = \frac{3}{2}$

$m_2 = \frac{3}{2}$

|| or co

$\frac{8-7}{3-0} = \frac{1}{3}$

Example 4: If the line through (x, -3) and (3, 1) is perpendicular to the line through (x, -3) and (-1, -2), find x.

$$m_1 \cdot m_2 = -1$$

$$\frac{4}{3-x} \cdot \frac{1}{-1-x} = -1$$

$$\frac{4}{(3-x)(-1-x)} = -1$$

$$\frac{4}{x^2-2x-3} = -1$$

$$-4 = x^2-2x-3$$

$$0 = x^2-2x+1$$

$$0 = (x-1)^2 \quad x=1$$