4.1 - The Standard Form For an Equation of a Circle

Definition

A circle is the set of all points in a plane at a fixed positive distance (radius) from a fixed point (center)

Theorem 4.1

A circle with center (h, k) and radius r has equation:

$(x - h)^2 + (y - k)^2 = r^2$

Think of the difference between $y = mx + b$ and $Ax + By + C = 0$. The slope intercept form readily shows you the slope and intercept of the line while the standard, or general form, is considered proper. Same goes for circles. While $(x - h)^2 + (h - k)^2 = r^2$, the standard form, readily gives you the center and radius, another form, the general form is preferred.

Theorem 4.2

Every circle can be represented in the general form:

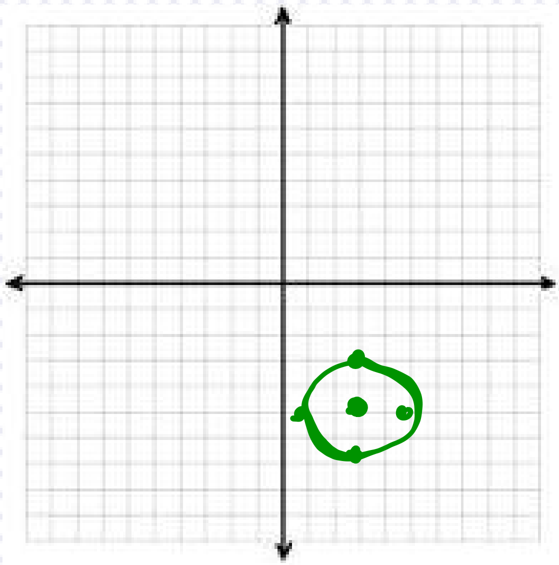
$Ax^2 + Ay^2 + Dx + Ey + F = 0$

$$\underline{A}x^2 + \cancel{Bxy} + \overset{A}{\cancel{C}}y^2 + Dx + Ey + F = 0$$

G.F. for conics

Example 1: Give an equation for the circle with center (3, -5) and radius 2. Graph this circle.

$$(x-3)^2+(y+5)^2=4$$



Example 2: Express $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ in standard form.

completing the square

x's y's = #

$$2x^2 - 2x + 2y^2 + 6y = 3$$

$$x^2 - x + \frac{1}{4} + y^2 + 3y + \frac{9}{4} = \frac{3}{2} + \frac{1}{4} + \frac{9}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 4$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$1x^2 + 2xy + y^2$$

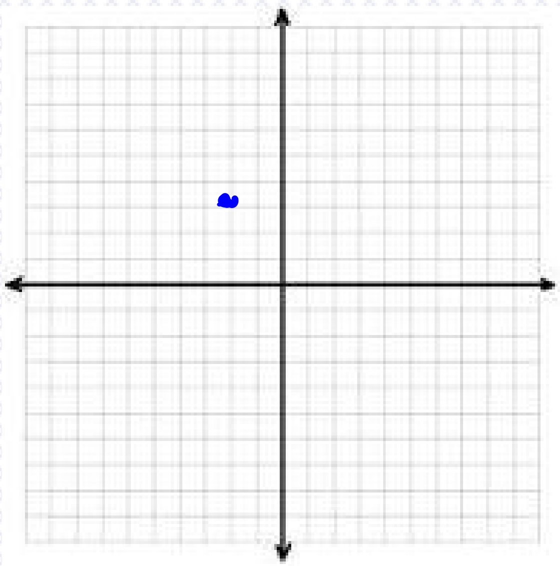
$$1x^2 + 8x + 16 \qquad (x+4)^2$$

$y = 4$

Example 3: Express $x^2 + y^2 + 4x - 6y + 13 = 0$ in standard form, then sketch the graph.

$$x^2 + 4x + 4 + y^2 - 6y + 9 = -13 + 4 + 9$$

$$(x+2)^2 + (y-3)^2 = 0$$



point

$\cdot \quad x \quad \cdot$

Example 4: Write the equation of a circle in general form that has (-3, 5) and (2, 4) as the endpoints of the diameter.

$d = \sqrt{25+1}$

Center

$\left(-\frac{1}{2}, \frac{9}{2}\right)$

$d = \sqrt{26}$

$r = \frac{\sqrt{26}}{2}$

$$\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{9}{2}\right)^2 = \left(\frac{\sqrt{26}}{2}\right)^2$$

$$x^2 + x + \frac{1}{4} + y^2 - 9y + \frac{81}{4} = \frac{26}{4}$$

$$x^2 + y^2 + x - 9y + \frac{56}{4} = 0$$

$$4x^2 + 4y^2 + 4x - 36y + 56 = 0$$

Example 5: Find the point(s) of intersection of $x^2 + y^2 + 4x - 12y + 6 = 0$ and $3x - 5y + 2 = 0$.

$x = \frac{5}{3}y - \frac{2}{3}$

$\left(\frac{5}{3}y - \frac{2}{3}\right)^2 + y^2 + 4\left(\frac{5}{3}y - \frac{2}{3}\right) - 12y + 6 = 0$

$\frac{25}{9}y^2 - \frac{20}{9}y + \frac{4}{9} + y^2 + \frac{20}{3}y - \frac{8}{3} - 12y + 6 = 0$

$25y^2 - 20y + 4 + 9y^2 + 60y - 24 - 108y + 54 = 0$

$34y^2 - 68y + 34 = 0$

$y^2 - 2y + 1 = 0$

$(y-1)^2 = 0$

$y = 1$

$(1, 1)$

$$x^2+y^2+5x+y-26=0$$
$$-\quad x^2+y^2+2x-y-15=0$$

$$3x+2y-11=0 \Rightarrow y=-\frac{3}{2}x-\frac{11}{2}$$

line w/ both intersection pts

$$x^2+\left(-\frac{3}{2}x-\frac{11}{2}\right)^2+5x+\left(-\frac{3}{2}x-\frac{11}{2}\right)-26=0$$

$$x=$$