14.2 - Euler Paths and Euler Circuits
The study of graph theory can be traced back to the eighteenth century when the people of the town of Konigsberg sought a solution to a popular problem. They had sections of the town connected with a series of seven bridges, the townspeople wondered if they could visit every part of the town without crossing the same bridge twice. Leonard Euler studied the problem, decided to represent the town as a graph with the areas being vertices and the bridges being edges. He then created graph theory.
The Bridges of Konigsberg Problem

Can you walk through town and cross all seven bridges without crossing any bridge twice?
Which of these designs can you trace without lifting your pencil AND without retracing any edge?

Note: Assume, each edge intersection is a vertex.
An Euler Path is a path that passes through each edge of a graph exactly one time.
An **Euler Circuit** is a circuit that passes through each edge of a graph exactly one time and ends where started.
An Euler Circuit is always a Euler Path, but ...

a Euler Path is not always a Euler Circuit.
Example 1: Find any Euler Paths or Euler Circuits

- (a) ABDEFA
- (b) DCABBD
- (c) Ø

Circuit

Path
Example 2: Determine the number of odd and even vertices then think back to the existence of either Euler Paths or Euler Circuits.
EULER'S THEOREM

1) A graph with no odd vertices (all even) has at least one Euler Path which is also a Euler Circuit. A Euler Circuit can be started at any vertex and will end at the same vertex.

2) A graph with exactly two odd vertices has at least one Euler Path but no Euler Circuits. Each Euler Path must start at an odd vertex and will end at the other.

3) A graph with more than two odd vertices has neither an Euler Path nor an Euler Circuit.
Example 3: Use Euler's Theorem to solve the Seven Bridges of Konigsberg Problem.
Example 4: Bob wants to visit each state in New England and cross each border once.

a) Is this possible, and if so in what order is it possible to visit each state?

b) If it is possible, can he start and end in the same state?
Example 5: Susan needs to visit a monitoring station in each state/province of the Pacific Northwest.

a) Is it possible for her to travel among these six states and cross each common border once?

b) Is it possible for her to start and end in the same state?
Example 6: You have wildlife observatories you need to check on in the Northwestern United States. Can you visit each state while crossing each state border only once? Can you start and stop in the same place? What routes might you take?
Is it possible to have a Euler Path or Circuit with the following graph? Why or why not?

No!
What is the minimum number of edges that need to be added to make an Euler Path or Circuit possible?
Fleury's Algorithm

A way to find Euler Paths and Circuits every time.

1) Determine if it is possible to make a path/circuit.

2) If a graph has no odd vertices, start anywhere, if a graph has an odd vertex start at an odd vertex.

3) Begin to trace the edges, and when given a choice never choose an edge that is a bridge (when taking into account remaining edges)

4) Continue until each edge of the entire graph has been traced once.
Example 7: In your subdivision the Neighborhood Association is planning to organize a crime stopper group where residents will take turn walking through the neighborhood making sure everything is ok. Can the residents start at one intersection and walk each street block exactly once, returning to where they started? If so in what order might they walk the streets?