3.1 - Statements and Logical Connectives

HISTORY

Aristotle is called the father of logic. Logic from his time period is called Aristotelian logic and it has been taught/studied for over 2000 years.

Gottfried Wilhelm Liebniz is the father of symbolic logic which we will be studying in this class.

Charles Dodgson a.k.a. Lewis Carroll incorporated many ideas from logic into his famous book Alice's Adventures in Wonderland.

Logic has been studied through the ages to exercise the mind's ability to reason. Understanding logic will enable you to think clearly, communicate effectively, make more convincing arguments, and develop patterns of reasoning that will help you in making decisions. It will also help you detect the fallacies in the reasoning or arguments of others such as advertisers and politicians.

LOGIC AND THE ENGLISH LANGUAGE

Connectives - the words and, or, if...then, both, etc.
- the words used to connect thoughts

Exclusive or - when the exclusive or is used, one or the other of the events can take place, but not both

ex. You may have a cup of soup or a half sandwich with your salad.

Inclusive or - when the inclusive or is used, one or the other, or both events can take place

ex. Your punishment is community service or a fine.

In this class the inclusive or is used unless stated otherwise.

STATEMENTS AND LOGICAL CONNECTIVES

Statement - A sentence that can be judged either true or false

Simple statement - a sentence that conveys only one idea.

ex. The Brooklyn Bridge goes over San Francisco Bay

Compound Statement - a sentence that conveys more than one idea which are linked by connectives.

ex. I will go to the birthday party or I will send a present.

QUANTIFIERS

Negation - changing a statement to its opposite meaning

The negation of a true statement is always false
The negation of a false statement is always true

Quantifiers - words that describe how much or how many

ex. all, none (or no), some

Form of statement	From of negation
All are	Some are not.
None are	Some are.
Some are	None are.
Some are not	All are.

SYMBOLIC LOGIC

Each idea is represented by a single letter, usually p, q, r, s, etc.

An idea that is a negation cannot be represented by a letter.

Not statements: ~p

And statements: these are conjucntions symbolized by p ∧ q

Or statements: these are disjunctions symbolized by p v q

If-then statements: these are conditional statements symbolized by p -> q

If and only if statements: these are biconditional statements symbolized by p <--> q

iff

GROUPING AND DOMINANCE OF CONNECTIVES

Grouping: the simple statements on the same side of the comma are to be grouped together within parentheses.

Table of Dominance:

Least Dominant	$\sim$ negation	Evaluate First
•	$\wedge, \vee$ con-/dis- junction	•
•	$\rightarrow$ conditional	•
Most Dominant	$\leftrightarrow$ biconditional	Evaluate Last

Example 1: Write the statements in symbolic form. Identify which letter is used for which idea.

a) Some students are athletes

p

b) Some dogs are not great danes.

$\sim p$

c) Maria will go to the circus or Maria will go to the zoo.

$p \vee q$

d)Dinner includes soup, and the salad or vegetable of the day

$p \wedge (q \vee r)$

e)Joseph is making breakfast and Mike is not setting the table

$p \wedge \sim q$

f)If Jennifer goes to the library then Jennifer will study.

$p \rightarrow q$

Example 1 Continued:

g) It is false that if June will go to the pool then she will swim.

$\sim(p \rightarrow q)$

h) Jorge will major in criminal science if and only if Jorge attends Georgetown.

$p \leftrightarrow q$

i) If Jorge's major is criminal justice, then Jorge is not enrolled in calculus or Jorge's major is engineering.

$p \rightarrow (\sim q \vee r)$

j) You are late in paying your rent, or if you have damaged the apartment then you may be evicted.

$p \vee (q \rightarrow r)$

k) It is false that the printer is working if and only if the ink cartridge is not correctly inserted.

$\sim(p \leftrightarrow \sim q)$

l)John is neither handsome nor rich.

$\sim p \wedge \sim q$

Example 2: Write each symbolic statement in words. Let
p: The sun is shining.
q: We go to Six Flags.
r: We ride roller coasters.

a) (p v q) ^ ~r

The sun is shining or we go to six flags, and we do not ride roller coasters.

b) ~p --> (q v r)

If The sun is not shining, then we go to six flags or we ride roller coasters

c) (q <--> p) ^ r

We go to six flags iff the sun is shining, and we ride roller coasters.

→ biggest out of ( )

Example 3: Add parentheses by using the dominance of connectives and the state if the statement is a negation, conjunction, disjunction, conditional, or biconditional.

a) (~p) --> (q) conditional

b) (~ p) ^ (~q) conjunction

c) (p v q) --> (r) conditional

d) (~ p) <--> (~ q --> r) biconditional

e) ~[(p ^ q) <--> (p v r)] negation