4.1 – Additive, Multiplicative, and Ciphered Systems of Numeration

Vocabulary

A **number** is a quantity, and it answers the question "How many?"

A **numeral** is a symbol used to represent a number.

A **system of numeration** consists of a set of numerals and a rule for combining the numerals to make numbers.
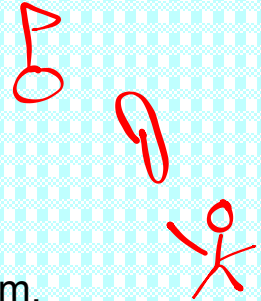
We use the **Hindu-Arabic system** for numeration.

ADDITIVE SYSTEMS

An additive system is one in which the number represented by a particular set of numerals is simply the sum of the values of the numerals. The additive system of numeration is one of the oldest and most primitive types of numeration systems.

The Egyptian System:

Hindu–Arabic Numerals	Egyptian Numerals	Description
1		Staff (vertical stroke)
10		Heel bone (arch)
100		Scroll (coiled rope)
1,000		Lotus flower
10,000		Pointing finger
100,000		Tadpole (or whale)
1,000,000		Astonished person

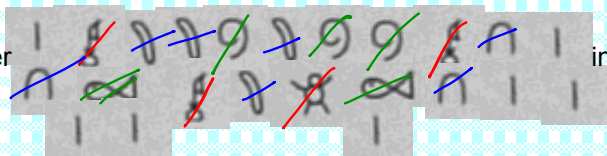The Egyptian System was the first additive system.

Order does not matter for the numerals.

45 symbols are needed to represent 99,999

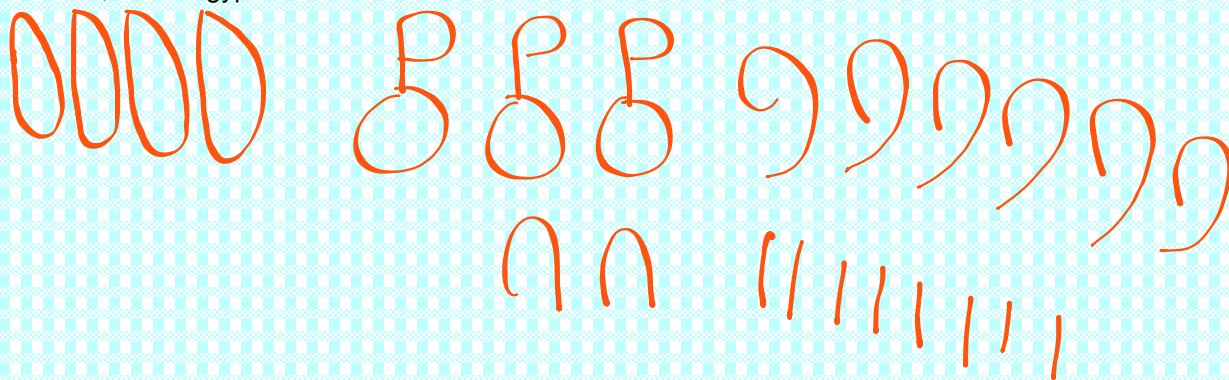Write the number [Egyptian numerals] in Hindu-Arabic.

$$1,000,000$$
$$200,000$$
$$40,000$$
$$3,000$$
$$300$$
$$+ \; 30 \atop 7} = 1,243,337$$

Write 43,628 in Egyptian.

The Roman Numeral System:

Roman numerals	I	V	X	L	C	D	M
Hindu–Arabic numerals	1	5	10	50	100	500	1000

$$C \quad I \quad X$$
$$160 \left(\underset{9}{1 - 10} \right)$$

Order does matter.

The Roman System uses the subtraction principle.

The numerals generally go from larger to smaller from left to right.

Starting from the left, we add each numeral unless its value is smaller than the value of the numeral to the right. In that case we subtract.

You may not have more than 3 of the same numeral. in a row

If you need 4 numerals, you simply use the larger numeral and then put the smaller numeral you couldn't originally use in front.

i.e. 4 = IIII which we can't do, but we can do V for 5 (the next larger numeral) with a I (the numeral we couldn't use) in front to make IV.

$$9 = VIIII \rightsquigarrow VIV \quad \text{or} \quad IX$$

A bar above a symbol or group of symbol indicates that the symbol(s) are to be multiplied by 1000.

$$\overline{XLVIII}$$
$$48,000$$

Write MMCCCLXII in Hindu-Arabic.

$1000 + 1000 + 100 + 100 + 100 + 50 + 10 + 1 + 1$

$2,362$

Write DCXLVI in Hindu-Arabic.

$500 + 100 + 10 + 50 + 5 + 1$

40

646

Write 289 in Roman.

CC L XXX VIIIII

IX

$$80 \neq XXC$$

no double
subtract

$$90 = XC$$

$$99 = IC \quad \text{or} \quad XCIX$$
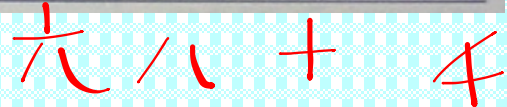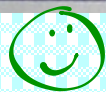
MULTIPLICATIVE SYSTEMS

Multiplicative Systems are more similar to Hindu-Arabic system.
No addition signs are needed to represent the number - it is implied.

The Chinese System:

Traditional Chinese numerals	零	一	二	三	四	五	六	七	八	九	十	百	千
Hindu–Arabic numerals	0	1	2	3	4	5	6	7	8	9	10	100	1000

The number is written vertically. The number on top will be 1 - 9, then below will be its corresponding power of 10.

You do not need to have a one on top of 10 when writing 11 - 19.

When more than one consecutive zero occurs, except at the end of a number, you need to write a zero, but only once if for consecutive zeros.

Samples:

$$406 = \begin{array}{l} 四 \\ 百 \end{array} \Big\} \quad 4 \times 100 = 400$$
$$\begin{array}{l} 零 \end{array} \Big\} \quad 0 \times 10 = 0$$
$$\begin{array}{l} 六 \end{array} \Big\} \; 6 \qquad = \quad 6$$

$$4006 = \begin{array}{l} 四 \\ 千 \end{array} \Big\} \quad 4 \times 1000 = 4000$$
$$\begin{array}{l} 零 \end{array} \Big\} \begin{array}{l} 0 \times 100, \\ 0 \times 10 \end{array} = \quad 0$$
$$\begin{array}{l} 六 \end{array} \Big\} \; 6 \qquad = \quad 6$$

$$460 = \begin{array}{l} 四 \\ 百 \end{array} \Big\} \quad 4 \times 100 = 400$$
$$\begin{array}{l} 六 \\ 十 \end{array} \Big\} \; 6 \times 10 = 60$$

$$4600 = \begin{array}{l} 四 \\ 千 \end{array} \Big\} \quad 4 \times 1000 = 4000$$
$$\begin{array}{l} 六 \\ 百 \end{array} \Big\} \; 6 \times 100 = 600$$

Write 538 in Chinese.

五
百
三
十
八

Write 7080 and 7008 in Chinese.

七
千
八
十
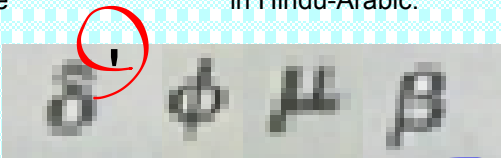
七
千
八

CIPHERED SYSTEMS

A ciphered numeration system is one in which there are numerals for numbers up to and including the base and for multiples of the base.

The Greek System

1	α	alpha	60	ξ	xi
2	β	beta	70	o	omicron
3	γ	gamma	80	π	pi
4	δ	delta	90	Q	koph*
5	ϵ	epsilon	100	ρ	rho
6	ζ	vau*	200	σ	sigma
7	ζ	zeta	300	τ	tau
8	η	eta	400	υ	upsilon
9	θ	theta	500	ϕ	phi
10	ι	iota	600	χ	chi
20	κ	kappa	700	ψ	psi
30	λ	lambda	800	ω	omega
40	μ	mu	900	$\pi\pi$	sampi*
50	ν	nu			

When a ' is placed above a number it multiplies that number by 1000.

Write in Hindu-Arabic.

δ′ φ μ β

4,000
500 40
2

4,542

Write 9432 in Iconic Greek.

θ′ν λ β

Homework