Indicate whether the statement is a simple or a compound statement. If it is a compound statement, indicate whether it is a negation, conjunction, disjunction, conditional, or biconditional by using both the word and its appropriate symbol.

1) The animal is a mammal if and only if it nurses its young.
   Compound statement, biconditional

2) The team leader has decided to take a vacation.
   Simple statement

3) It is false that whales are fish and bats are birds.
   Compound statement, negation

Convert the compound statement into words.

4) p = "Students are happy." q = "Teachers are happy." $\sim(p \vee \sim q)$
   It is false that the students are happy or the teachers are not happy.

5) p = "The food tastes delicious." q = "We eat a lot." r = "Nobody has dessert." $\sim q \vee (p \wedge r)$
   We do not eat a lot, or the food tastes delicious and nobody has dessert.

Add parentheses using the dominance of connectives and then indicate whether the statement is a negation, conjunction, disjunction, conditional, or biconditional.

6) $\sim p \wedge q \rightarrow \sim r$
   $(\sim p \wedge q) \rightarrow (\sim r)$, Conditional

7) $p \vee \sim q \rightarrow \sim r \wedge q$
   $(p \vee \sim q) \rightarrow (\sim r \wedge q)$, Conditional

8) $\sim[p \leftrightarrow r \vee q]$
   $\sim[p \leftrightarrow (r \vee q)]$, Negation

Select letters to represent the simple statements and write each statement symbolically by using parentheses then indicate whether the statement is a negation, conjunction, disjunction, conditional, or biconditional.

9) If people drive small cars then people will use less fuel and the ozone hole will not expand.
   $P \rightarrow (Q \wedge \sim R)$; Conditional - parentheses added by dominance of connectives

10) The lights are on if and only if it is not midnight or it is wintertime.
   $P \leftrightarrow (\sim Q \vee R)$; Biconditional -parentheses added by dominance of connectives

11) If tomorrow is not Saturday then today is Friday if and only if tomorrow is Saturday.
   $(\sim P \rightarrow Q) \leftrightarrow P$; Biconditional -parentheses added by dominance of connectives

Let p represent a true statement, while q and r represent false statements. Find the truth value of the compound statement.

12) $\sim[(\sim p \wedge \sim q) \vee \sim q]$
   $\sim[(\sim T \wedge \sim F) \vee \sim F]$
   $\sim[(F \wedge T) \vee T]$
   $\sim[(F) \vee T]$
   $\sim[T]$
   F

13) $\sim(p \wedge q) \wedge (r \vee \sim q)$
   $\sim(T \wedge F) \wedge (F \vee \sim F)$
   $\sim(T \wedge F) \wedge (F \vee T)$
   $\sim(F) \wedge (T)$
   $T \wedge (T)$
   T

14) ~(~p ∧ ~q) ∨ (~r ∨ ~p)

~(~T ∧ ~F) ∨ (~F∨ ~T)
~(F ∧ T) ∨ (T∨ F)
~(F) ∨ (T)
T ∨ (T)
T

Given p is true, q is true, and r is false, find the truth value of the statement.

15) (q ∨ r) ↔ (p ∧ q)

(T ∨ F) ↔ (T ∧ T)
(T) ↔ (T)
T

16) (~p ↔ ~q) ∧ (p ↔ ~r)

(~T ↔ ~T) ∧ (T ↔ ~F)
(F ↔ F) ∧ (T ↔ T)
(T) ∧ (T)
T

17) (~p ∧ q) ↔ ~r

(~T ∧ T) ↔ ~F
(F ∧ T) ↔ T
(F) ↔ T
F

18) [(~p → r) ∧ (~p ∨ q)] → r

[(~T → F) ∧ (~T ∨ T)] → F
[(F → F) ∧ (F ∨ T)] → F
[(T) ∧ (T)] → F
[T]→ F
F

Determine the truth value for each simple statement. Then, using the truth values, give the truth value of the compound statement.

19) The capital of Illinois is Chicago and Iowa is west of the Mississippi River, or Tallahassee is the capital of Florida.

(P ∧ Q) ∨ R
(F ∧ T) ∨ T
(F) ∨ T
T

20) In Region X, people are spending more on leisure activities. Annual per capita spending in dollars:

	1970	1999
Movies	32	182
Dining Out	97	486
Vacations	505	1535
Sporting Goods	105	346

The average person in Region X spent $182 on the movies in 1999, and the average person in Region X spent $105 on sporting goods in 1999.

P ∧ Q
T ∧ F
F

21) 7 x 2 = 18 if and only if 7 + 5 = 12.

P ↔ Q
F ↔ T
F

22) One dollar has the same value as 15 dimes and one quarter has the same value as 25 pennies, or one dime has the same value as 5 nickels.

(P ∧ Q) ∨ R
(F ∧ T) ∨ F
(F) ∨ F
F

Construct a truth table for the statement.

23) ~(p ∨ q) ∧ ~(q ∧ p)

P	Q	~	(P	∨	Q)	∧	~	(Q	∧	P)
T	T	F	T	T	T	F	F	T	T	T
T	F	F	T	T	F	F	T	F	F	T
F	T	F	F	T	T	F	T	T	F	F
F	F	T	F	F	F	T	T	F	F	F

24) (r ∧ p) ∧ (~ p ∨ q)

P	Q	R	(R	∧	P)	∧	(~P	∨	Q)
T	T	T	T	T	T	T	F	T	T
T	T	F	F	F	T	F	F	T	T
T	F	T	T	T	T	F	F	F	F
T	F	F	F	F	T	F	F	F	F
F	T	T	T	F	F	F	T	T	T
F	T	F	F	F	F	F	T	T	T
F	F	T	T	F	F	F	T	T	F
F	F	F	F	F	F	F	T	T	F

25) ~[p ↔ (~q)]

P	Q	~	[P	↔	~Q]
T	T	T	T	F	F
T	F	F	T	T	T
F	T	F	F	T	F
F	F	T	F	F	T

26) ~p → (~p ∧ q)

P	Q	~P	→	(~P	∧	Q)
T	T	F	T	F	F	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	F	T	F	T	F	F

27) ~(p ∧ r) → (p → (~ q ∧ r))

P	Q	R	~	(P	∧	R)	→	(	P	→	(~Q	∧	R)	)
T	T	T	F	T	T	T	T		T	F	F	F	T	
T	T	F	T	T	F	F	F		T	F	F	F	F	
T	F	T	F	T	T	T	T		T	T	T	T	T	
T	F	F	T	T	F	F	F		T	F	T	F	F	
F	T	T	T	F	F	T	T		F	T	F	F	T	
F	T	F	T	F	F	F	T		F	T	F	F	F	
F	F	T	T	F	F	T	T		F	T	T	T	T	
F	F	F	T	F	F	F	T		F	T	T	F	F	

Determine whether the statement is a self-contradiction, an implication, a tautology, or none of these.

28) [(p ∨ q) ∨ r] → [~r ∧ (p ∧ q)]

P	Q	R	[	(P	∨	Q)	∨	R	]	→	[	~R	∧	(P	∧	Q)	]
T	T	T		T	T	T	T	T		F		F	F	T	T	T	
T	T	F		T	T	T	T	F		T		T	T	T	T	T	
T	F	T		T	T	F	T	T		F		F	F	T	F	F	
T	F	F		T	T	F	T	F		F		T	F	T	F	F	
F	T	T		F	T	T	T	T		F		F	F	F	F	T	
F	T	F		F	T	T	T	F		F		T	F	F	F	T	
F	F	T		F	F	F	T	T		F		F	F	F	F	F	
F	F	F		F	F	F	F	F		T		T	F	F	F	F	

This statement is neither as it is not all false or all true.

29) (p ∨ q) ∧ ~q

P	Q		(P	∨	Q)	∧	~Q
T	T		T	T	T	F	F
T	F		T	T	F	T	T
F	T		F	T	T	F	F
F	F		F	F	F	F	T

This statement is neither as it is not all false or all true.

30) ~[(p → q) ∧ (q → p)] → ~(p ↔ q)

P	Q		~	[	(P	→	Q)	∧	(Q	→	P)	]	→	~	(P	↔	Q)
T	T		F		T	T	T	T	T	T	T		T	F	T	T	T
T	F		T		T	F	F	F	F	T	T		T	T	T	F	F
F	T		T		F	T	T	F	T	F	F		T	T	F	F	T
F	F		F		F	T	F	T	F	T	F		T	F	F	T	F

This statement is an implication as the conditional statement is all true.

Use DeMorgan's laws or a truth table to determine whether the two statements are equivalent.

31) ~(~p → q), p ∨ ~q

P	Q		~	(~P	→	Q)		P	∨	~Q
T	T		F	F	T	T		T	T	F
T	F		F	F	T	F		T	T	T
F	T		F	T	T	T		F	F	F
F	F		T	T	F	F		F	T	T

Not Equivalent – the two answer columns are different!

32) ~(p ∧ q), ~p ∧ ~q

P	Q		~	(P	∧	Q)		~P	∧	~Q
T	T		F	T	T	T		F	F	F
T	F		T	T	F	F		F	F	T
F	T		T	F	F	T		T	F	F
F	F		T	F	F	F		T	T	T

Not Equivalent – the two answer columns are different!

Also, by DeMorgan's law we know they are not equivalent as the conjunction did not flip when the negation was distributed.

33) ~(p ∨ q) → r, (~p ∧ ~q) → r

P	Q	R		~	(P	∨	Q)	→	R		(~P	∧	~Q)	→	R
T	T	T		F	T	T	T	T	T		F	F	F	T	T
T	T	F		F	T	T	T	T	F		F	F	F	T	F
T	F	T		F	T	T	F	T	T		F	F	T	T	T
T	F	F		F	T	T	F	T	F		F	F	T	T	F
F	T	T		F	F	T	T	T	T		T	F	F	T	T
F	T	F		F	F	T	T	T	F		T	F	F	T	F
F	F	T		T	F	F	F	T	T		T	T	T	T	T
F	F	F		T	F	F	F	F	F		T	T	T	F	F

Equivalent – the two answer columns are the same!

34) (p ∨ q) ∨ r, p ∨ (q ∨ r)

P	Q	R		(P	∨	Q)	∨	R		P	∨	(Q	∨	R)
T	T	T		T	T	T	T	T		T	T	T	T	T
T	T	F		T	T	T	T	F		T	T	T	T	F
T	F	T		T	T	F	T	T		T	T	F	T	T
T	F	F		T	T	F	T	F		T	T	F	F	F
F	T	T		F	T	T	T	T		F	T	T	T	T
F	T	F		F	T	T	T	F		F	T	T	T	F
F	F	T		F	F	F	T	T		F	T	F	T	T
F	F	F		F	F	F	F	F		F	F	F	F	F

Equivalent – the two answer columns are the same!

Write an equivalent sentence for the statement.

35) It is not true that you are a day late and a dollar short.

(*Hint:* Use De Morgan's laws.)

Problem: ~(P ∧ Q)

DeMorgan's Equivalence: ~P ∨ ~Q

New Sentence: You are not a day late or you are not a dollar short.

36) You do not give your rain coat to the doorman or he will give you a dirty look.

(*Hint:* Use the fact that p → q is equivalent to ~p ∨ q.)

Problem: ~P ∨ Q

Equivalence: P → Q

New Sentence: If you give your rain coat to the doorman then he will give you a dirty look.

37) If it is raining, you take your umbrella.

(*Hint:* Use the fact that p → q is equivalent to ~p ∨ q.)

Problem: P → Q

Equivalence: ~P ∨ Q

New Sentence: It is not raining or you take your umbrella.

38) If you are thoughtless then you are rude, and if you are rude then you are thoughtless.

(*Hint:* Use the fact that (p → q) ∧ (q → p) is equivalent to p ↔ q.)

Problem: (P → Q) ∧ (Q → P)

Equivalence: P ↔ Q

New Sentence: You are thoughtless if and only if you are rude.

Re-write the conditional statements as indicated.

39) If I pass, I'll party. Contrapositive

If I do not party then I did not pass.

40) If x = 4, then x^2 = 16. Converse

If $x \neq 4$, then $x^2 \neq 16$.

41) If the moon is out, then we will start a campfire and we will roast marshmallows. Inverse

If we start a campfire and we roast marshmallows then the moon is out.

Write the contrapositive of the statement. Then use the contrapositive to determine whether to conditional statement is true or false.

42) If the triangle is not equilateral, then the three sides of the triangle are not equal.

Contrapositive: If the three sides of a triangle are equal, then the triangle is equilateral.

True

43) If $\frac{1}{n}$ is not an integer, then n is not an integer.

Contrapositive: If n is an integer then $\frac{1}{n}$ is an integer.

False

Determine which, if any, of the three statements are equivalent.

44)

I) Jan is well or Jan is still recovering.
II) If Jan is still recovering, then Jan is not well.
III) If Jan is well, then Jan is not still recovering.

Symbolic Statements:

I) $P \vee Q$
II) $Q \to \sim P$
III) $P \to \sim Q$

Statements II and III are equivalent. We can say that II is the conditional and III is then the contrapositive (switch the statements and negate to find equivalent conditional statements).

Another way: II would also be equivalent to $\sim Q \vee \sim P$ while III would also be equivalent to $\sim P \vee \sim Q$. We see that these are the same disjunctive statement, thus II and III are equivalent and cannot be the same as I.

45)

I) If it is sunny and the pool is open, then I will go swimming.
II) If I do not go swimming, then it it is not the case that it is sunny or the pool is open.
III) It is sunny and the pool is open, or I will go swimming.

Symbolic Statements:

I) $(P \wedge Q) \to R$
II) $\sim R \to \sim(P \vee Q)$
III) $(P \wedge Q) \vee R$

None of them are equivalent.

I and II are almost equivalent if they wouldn't have switched the and to an or for the same reason II and II are equivalent in #44.

I and III cannot be equivalent as you cannot change conditional to disjunction.

II and III would be equivalent if the first part would have had a negation outside the parentheses (see #36)