3.1 – Statements and Logical Connectives

1) List the dominance of connectives from the most dominant to the least dominant.

Indicate whether the statement is a simple statement or a compound statement. If it is a compound statement indicate whether it is a negation, conjunction, disjunction, conditional or Biconditional by using both the word and the symbol.

2) The sun is shining and the air is crisp.

3) If the electricity goes out then the standard telephone will still work.

4) The hurricane did $400,000 worth of damage to DeSoto County.

5) If Cathy Smith walks 4 miles today then she will be sore tomorrow.

6) It is false that if John Wubben fixes your car then you will need to pay him in cash.

Write the negation of the statement.

7) No stock mutual funds have guaranteed yields.

8) Some turtles do not have claws.

9) All horses have manes.

10) Some pine trees do not produce pinecones.

11) No one likes asparagus.

Write the statement in symbolic form if:
        p: The tent is pitched.
        q: The bonfire is burning.

12) The tent is not pitched.

13) The bonfire is not burning if and only if the tent is not pitched.

14) If the tent is not pitched then the bonfire is not burning.

Write the statement in symbolic form if:
     p: The charcoal is hot.
     q: The chicken is on the grill.

15) Neither is the charcoal hot nor is the chicken on the grill.

16) The charcoal is not hot, but the chicken is on the grill.

17) It is false that the charcoal is hot and the chicken is on the grill.

Write the statement in words if:
     p: Firemen work hard.
     q: Firemen wear red suspenders.

18) ~q

21) ~(q v p)

19) p ^ q

22) p v q

20) ~p → q

Write the statement in symbolic form if:
     p: The temperature is 90^oF.
     q: The air conditioner is working.
     r: The apartment is hot.

23) The apartment is hot if and only if the temperature is not 90^oF, or the air conditioner is not working.

24) If the apartment is hot and the air conditioner is working, then the temperature is 90^oF.

25) It is false that if the apartment is hot then the air conditioner is not working.

26) The apartment is hot or the air conditioner is not working, if and only if the temperature is 90^oF.

Write the statement in words if:
  p: The water is 70°F.
  q: The sun is shining
  r: We go swimming

27) (p v q) ^ ~r

28) ~p → (q v r)

29) (q ←→p) ^r

30) (p →r) ^ p

31) (p ^ q) v r

Add parentheses to each statement by using the dominance of connectives and indicate whether the statement is a negation, conjunction, disjunction, conditional or Biconditional.

32) ~p ^ q ←→ ~q

33) p v q

34) q →p ^ ~r

35) ~[p → q v r]

36) ~[r ^ ~q →q ^ r]

37) ~q ←→ ~q →r

38) ~q ^ ~r

39) r ^ ~q → q v ~p

40) r v q → ~p

41) q → p ←→p → q

Select letters to represent the simple statements and write each statement symbolically by using parentheses. Then indicate whether the statement is a negation, conjunction, disjunction, conditional or biconditional.

42) Ruth Bignel retired, but she did not start her concrete business.

43) It is false that if your speed is below the speed limit then you will not get pulled over.

44) If the food has fiber or the food has vitamins, then you will be healthy.

45) You may take this course if an only if you did not fail the previous course or you passed the placement test.

46) The classroom is empty if and only if it is the weekend, or it is 7am.

47) If the car has the gas and the battery is charged then the car will start.

3.2 & 3.3 – Truth Tables for Negation, Conjunction, Disjunction, Conditional and Biconditional

Answer the following questions regarding the construction of truth tables.

48) How many distinct cases must be listed in a truth table that contains:
   a. Two simple statements?

   b. Three simple statements?

49) Under what circumstances is a conjunction false?

50) Under what circumstances is a disjunction false?

51) Under what circumstances is a conditional false?

52) Under what circumstances is a Biconditional false?

Determine the truth value of the statement if:
   a) **P is true, Q is false, and R is true.**
   b) **P is false, Q is true, and R is true.**

53) ~p v (q ^ r)

57) (q ^ ~p) ←→~r

54) (~p ^ r) ^ q

58) [(~r→~q) v (p ^~r)]→q

55) (p v ~q) ^ ~(p ^ ~r)

59) (p^r) ←→(p v ~q)

56) P →(~q ^ r)

60) ~[(p v q) ←→ (p → ~r)]

Write the statement in symbolic form and construct a truth table.
   61) Meetings are dull and teaching is fun.

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62) Bob will get a haircut, but he will not shave his beard.

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63) It is false that Jasper Adams is a tutor and Mark Russo is a secretary.

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64) The copier is out of toner, or the lens is dirty or the corona wires are broken.

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65) I am hungry, and I want to eat a healthy lunch and I do not want to eat in a hurry.

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66) If I drink a glass of water, then I will have a better complexion and I will sleep better.

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67) If Mary Andrews does not send me an e-mail then we can call her, or we can write to Mom.

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68) The goalie will make the save if and only if the stopper is in position, or the forward cannot handle the ball.

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Construct a truth table for the following.

69) p v ~q

70) ~p ^ ~q

71) r v (p ^ ~q)

72) (p v q) v ~q

73) q →(p →~q)

74) (p v q) ←→(p ^ q)

75) ~[r ^ (q v ~p)] ←→ ~p →r

76) ~[(~r →~q) v (p ^ ~r)] → ~q

Determine the truth value for each simple statement then use those truth values to determine the truth value of the compound statement. You may have to use a reference source such as the internet or an encyclopedia.

77) 3 + 5 = 4 + 4 or 10 − 9 = 9 − 10

78) Elvis Presley was a singer or chickens can swim.

79) Cal Ripkin Jr. played football or George Bush was the prime minister of England, and Colin Powell was in the Army.

80) Alaska is the 50th state or Hawaii is a group of islands, and Atlanta is the capital of Alabama.

81) If 10 + 5 = 15, then 56/7 = 8.

82) Dell makes computers, if and only if Gateway makes computers or Cannon makes printers.

83) Valentine's Day is in February or President's Day is in March, and Thanksgiving is in November.

84) Spike Lee is a movie director, or if Halle Berry is a school-teacher then George Clooney is a circus clown.

Determine whether the statement is a tautology, self-contradiction, or neither.

85) p → ~q

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86) (p v q) ←→ ~p

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87) $[(p \rightarrow q) \vee r] \leftrightarrow [(p \wedge q) \rightarrow r]$

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88) $(p \wedge \sim q) \rightarrow q$

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Determine whether the statement is an implication.

89) $p \rightarrow (\sim q \wedge r)$

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90) (q ^ p) →(p v q)

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91) [(p v q) ^ r] →(p v q)

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3.4 – Equivalent Statements

Inverse, Converse, Contrapositive & Logical Equivalence.

92) Which of the following are equivalent statements?
   a. The converse.
   b. The contrapositive.
   c. T he inverse.
   d. The conditional.

93) For a statement of the form p→q, symbolically indicate the forms of the:
   a. Converse

   b. Contrapositive

   c. Inverse

94) Write a disjunctive statement that is logically equivalent to p →q.

95) Write a conjunction involving two conditional statements that is logically equivalent to p ↔q.

96) Write the converse, inverse, and contrapositive of the following statements:
   a. If the book is interesting then I will finish the book in 1 week.
      i. Converse:

      ii. Inverse:

      iii. Contrapositive:

   b. If you finish your homework, then you can watch televisions.
      i. Converse:

      ii. Inverse:

      iii. Contrapositive:

Use DeMorgan's laws to determine whether the statements are equivalent.
   97) ~p v ~q;   ~(p ^ q)

   98) ~(p v q);   ~p ^ ~q

   99) ~(p ^ q);   ~p ^ ~q

   100) ~(p ^ q);   ~p ^ q

Use DeMorgan's laws to determine whether the statements are equivalent.

101) ~(p v ~q);   ~p ^ q

102) ~(p ^ q);   ~(q v ~p)

103) (~p v ~q) →r;   ~(p ^ q) →r

104) q →~(p ^ ~r);   q→~p v r

105) ~(p→~q);   p ^ q

106) ~(~p→q);   ~p ^~q

Use DeMorgan's laws to write an equivalent statement for the sentence.

107) It is false that the Mississippi River runs through Ohio or the Ohio River runs through Mississippi.

108) The snowmobile was neither an Arctic Cat nor was it a Ski-Do.

109) The pot roast is hot, but it is not well done.

110) If we go to Cozumel, then we will go snorkeling or we will not go to Senior Frogs.

111) The hotel does not have a weight room or the conference center does not have an auditorium.

Use the fact that p→q is equivalent to ~p v q to write an equivalent form of the given statement.

112) If you drink a glass of orange juice, then you will get a full day supply of folic acid.

113) Bob the Tomato visited the nursing home or he did not visit the Cub Scout meeting.

114) It is false that if the plumbers meet in Kansas City then the Rainmakers will provide the entertainment.

Use the face that (p→q) ^ (q→p) is equivalent to p←→q to write an equivalent form of the given statement.

115) If it is cloudy then the front is coming through and if the front is coming through then it is cloudy.

116) The chemistry teacher teaches mathematics if and only if there is a shortage of mathematics teachers.

117) If Model Road is closed then we use Kirkwood Road and if we use Kirkwood Road then Model Road is closed.

Use a truth table (if necessary) to determine whether the statements are equivalent.

118) p →q;   ~p v q

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119) ~(p←→q) ;   [~(p→q)] ^ [~(q→p)]

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120) (p ^ q) ^ r;   p ^ (q ^ r)

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121) $(p \rightarrow q) \wedge (q \rightarrow p);$ $p \leftrightarrow q$

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122) $\sim p \rightarrow q;$ $p \wedge q$

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123) $\sim q \rightarrow (p \wedge r);$ $\sim(p \vee r) \rightarrow q$

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124) $(p \rightarrow q) \wedge (q \rightarrow r);$ $(p \rightarrow q) \rightarrow r$

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