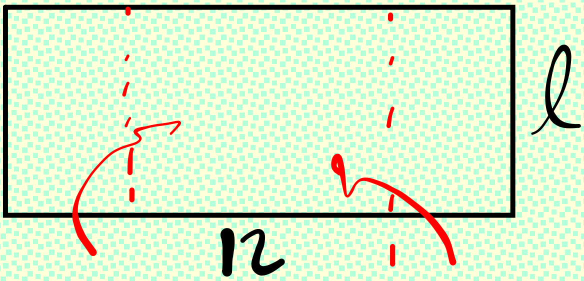
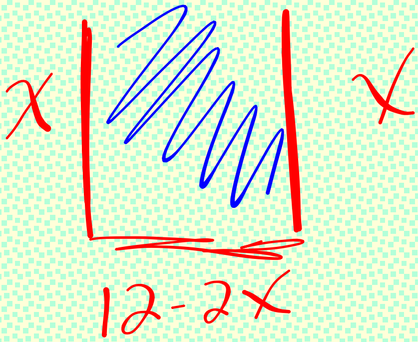
4.6 - Optimization Problems

Jo - Z
B - M
C - S

R - Jv
ladies

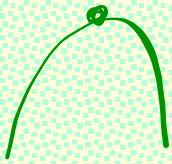
Example 1: A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?





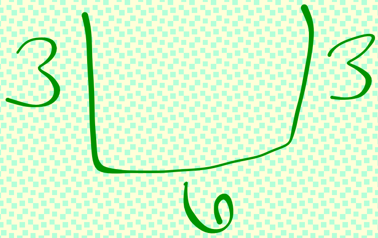
Optimize: $x(12-2x)$ (max.)

$= 12x-2x^2$

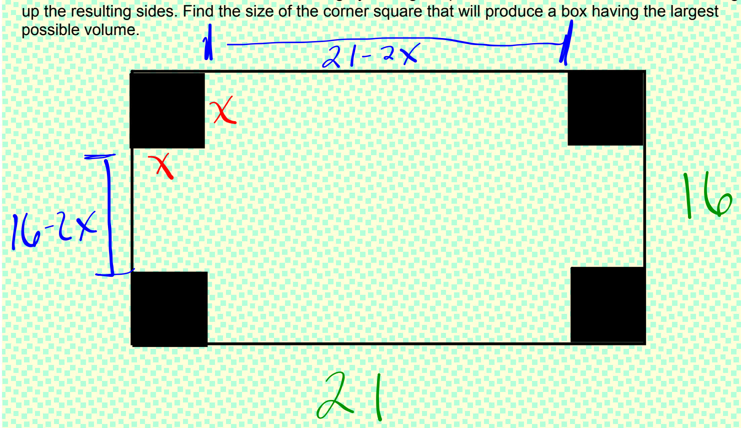


derivative: $12-4x$

CN: $\boxed{3}$ maximum



Example 2: An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 inches wide and 21 inches long by cutting a square from each corner and then bending up the resulting sides. Find the size of the corner square that will produce a box having the largest possible volume.



Max Volume

$$V = (16-2x)(21-2x)(x)$$

$$V = (16-2x)(21x-2x^2)$$

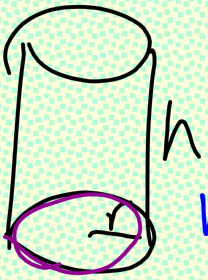
$$V' = (16-2x)(21-4x) + (21x-2x^2)(-2)$$

$$V' = 12x^2 - 148x + 336$$

CN: 3 or 28/3

CN	Volume
3	450
~~$\frac{28}{3}$~~	~~$-\frac{1568}{27}$~~

Example 3: A circular cylindrical metal container, open at the top, is to have a capacity of 24π in^3. The cost of the material used for the bottom of the container is 15 cents per square inch and that of the material used for the curved part is 5 cents per square inch. If there is no waste of material find the dimensions that will minimize the cost of the material.



Constraint

$$V = 24\pi = \pi r^2 h$$

$$24 = r^2 h$$

$$h = \frac{24}{r^2}$$

15¢ botm $\rightarrow \pi r^2$ (surface area)

5¢ side $\rightarrow 2\pi r h$ (surface area)

$$\text{Cost} = .15\pi r^2 + .05(2\pi r h)$$

$$C = .15\pi r^2 + .05\left(2\pi r \frac{24}{r^2}\right)$$

$$C = .15\pi r^2 + 2.4\pi r^{-1}$$

$$C' = .3\pi r - 2.4\pi r^{-2}$$

$$C' = r^{-2}(.3\pi r^3 - 2.4\pi)$$

$$r = 0 \text{ or } r = \sqrt[3]{\frac{2.4}{.3}}$$

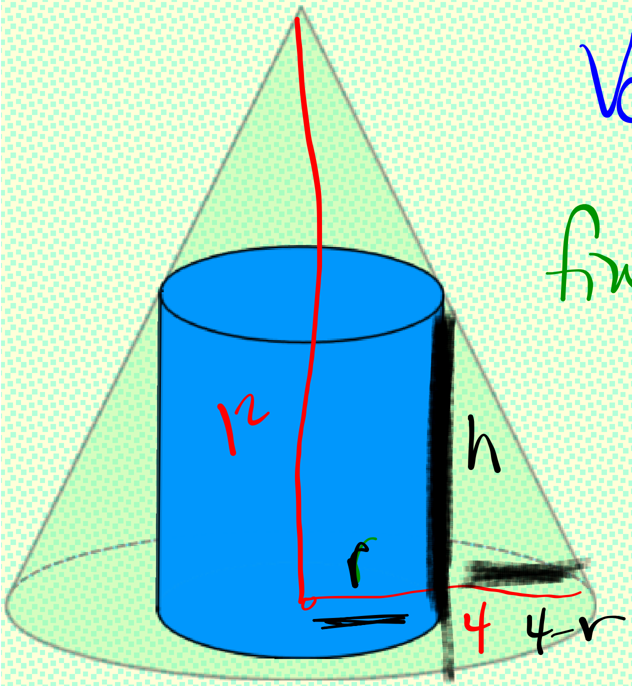
$$r = \sqrt[3]{8}$$

$$r = 2$$

$$h = \frac{24}{4}$$

$$h = 6$$

Example 4: Find the maximum volume of a right circular cylinder that can be inscribed in a cone of altitude 12 centimeters and base radius 4 centimeters, if the axes of the cylinder and cone coincide.



$V_{cyl} = \pi r^2 h \rightarrow$ max

find formula for r or h

$$\frac{12}{4} = \frac{h}{4-r}$$

$$3 = \frac{h}{4-r}$$

$$h = 12 - 3r$$

$$V_{cyl} = \pi r^2(12-3r)$$

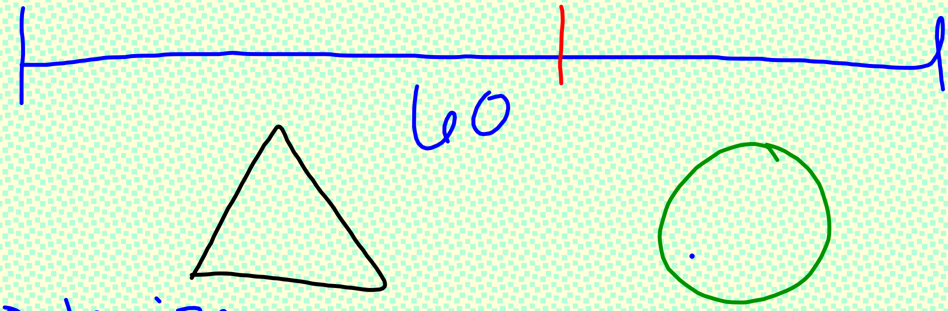
$$V = 12\pi r^2 - 3\pi r^3$$

$$V' = 24\pi r - 9\pi r^2$$

$$= 3\pi r(8-3r)$$

$$r = 0 \text{ or } \boxed{\frac{8}{3}}$$

Example 7: A wire 60 inches long is to be cut into two pieces. One of the pieces will be bent into the shape of a circle and the other into the shape of an equilateral triangle. Where should the wire be cut so that the sum of the areas of the circle and triangle is minimized? Maximized?



optimize
max Area $= \frac{\sqrt{3}}{4}s^2 + \pi r^2$

constraint fxn: $3s + 2\pi r = 60$

$r = \frac{60-3s}{2\pi}$

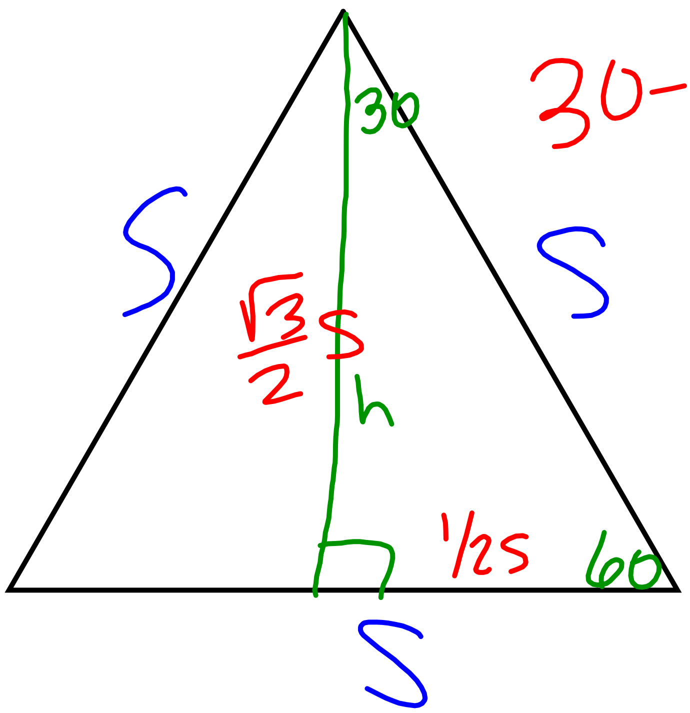
$$\text{Area} = \frac{\sqrt{3}}{4}s^2 + \pi\frac{(60-3s)^2}{4\pi^2}$$

$$= \frac{\sqrt{3}}{4}s^2 + \frac{3600-360s+9s^2}{4\pi}$$

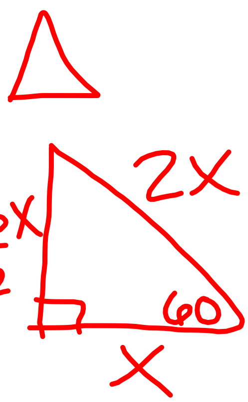
$$A' = \frac{\sqrt{3}}{2}s + \frac{-360}{4\pi} + \frac{18s}{4\pi}$$

. . .

finish



30-60-90



$$\frac{1}{2}bh = \frac{1}{2}(S)\left(\frac{\sqrt{3}}{2}S\right)$$

$$= \frac{\sqrt{3}}{4}S^2$$

PROJECT

optomize

$SA = 2\pi r^2 + h(2\pi r)$

2circ + rect

$l \cdot w$

constraint

$V = \pi r^2 h$

① measure height & circumference

② from circ. approx r. to 3 decimal places

③ Calc volume

Say. . $h = \frac{V}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right)$

simp

derive (V is const.)